9.1 Translate Figures and Use Vectors

Before You used a coordinate rule to translate a figure.

Now You will use a vector to translate a figure.

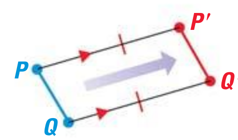
Why? So you can find a distance covered on snowshoes, as in Exs. 35–37.

Key Vocabulary
- **image**
- **preimage**
- **isometry**
- **vector**
  initial point, terminal point, horizontal component, vertical component
- **component form**
- **translation,** *p. 272*

In Lesson 4.8, you learned that a *transformation* moves or changes a figure in some way to produce a new figure called an **image**. Another name for the original figure is the **preimage**.

Recall that a *translation* moves every point of a figure the same distance in the same direction. More specifically, a translation maps, or moves, the points P and Q of a plane figure to the points P' (read "P *prime*") and Q', so that one of the following statements is true:

- $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$, or
- $PP' = QQ'$ and $\overline{PP'}$ and $\overline{QQ'}$ are collinear.



EXAMPLE 1 Translate a figure in the coordinate plane

Graph quadrilateral $ABCD$ with vertices $A(-1, 2)$, $B(-1, 5)$, $C(4, 6)$, and $D(4, 2)$. Find the image of each vertex after the translation $(x, y) \to (x + 3, y - 1)$. Then graph the image using prime notation.

Solution

USE NOTATION
You can use *prime notation* to name an image. For example, if the preimage is $\triangle ABC$, then its image is $\triangle A'B'C'$, read as *"triangle A prime, B prime, C prime."*

First, draw $ABCD$. Find the translation of each vertex by adding 3 to its x-coordinate and subtracting 1 from its y-coordinate. Then graph the image.

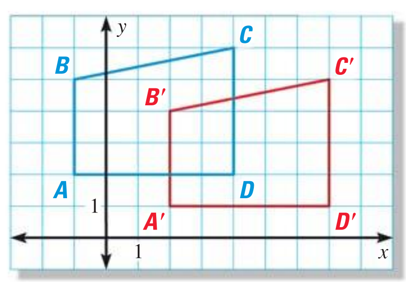
$$(x, y) \to (x + 3, y - 1)$$

$A(-1, 2) \to A'(2, 1)$

$B(-1, 5) \to B'(2, 4)$

$C(4, 6) \to C'(7, 5)$

$D(4, 2) \to D'(7, 1)$



✓ GUIDED PRACTICE for Example 1

1. Draw $\triangle RST$ with vertices $R(2, 2)$, $S(5, 2)$, and $T(3, 5)$. Find the image of each vertex after the translation $(x, y) \to (x + 1, y + 2)$. Graph the image using prime notation.

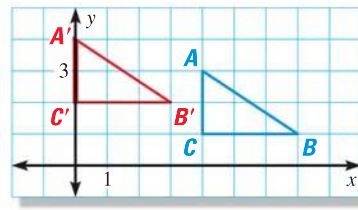
2. The image of $(x, y) \to (x + 4, y - 7)$ is $\overline{P'Q'}$ with endpoints $P'(-3, 4)$ and $Q'(2, 1)$. Find the coordinates of the endpoints of the preimage.

ISOMETRY An **isometry** is a transformation that preserves length and angle measure. Isometry is another word for congruence transformation (page 272).

EXAMPLE 2 Write a translation rule and verify congruence

READ DIAGRAMS
In this book, the preimage is always shown in blue, and the image is always shown in red.

Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the transformation is an isometry.



Solution

To go from A to A', move 4 units left and 1 unit up. So, a rule for the translation is $(x, y) \rightarrow (x - 4, y + 1)$.

Use the SAS Congruence Postulate. Notice that $CB = C'B' = 3$, and $AC = A'C' = 2$. The slopes of $\overline{CB}$ and $\overline{C'B'}$ are 0, and the slopes of $\overline{CA}$ and $\overline{C'A'}$ are undefined, so the sides are perpendicular. Therefore, $\angle C$ and $\angle C'$ are congruent right angles. So, $\triangle ABC \cong \triangle A'B'C'$. The translation is an isometry.

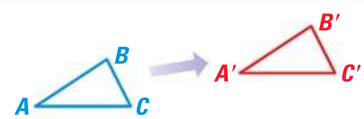
✓ GUIDED PRACTICE for Example 2

3. In Example 2, write a rule to translate $\triangle A'B'C'$ back to $\triangle ABC$.

THEOREM — *For Your Notebook*

THEOREM 9.1 Translation Theorem

A translation is an isometry.

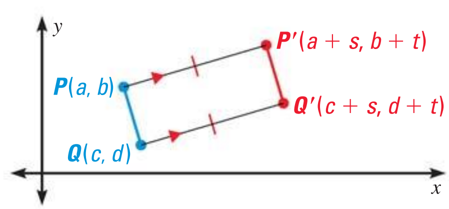


$\triangle ABC \cong \triangle A'B'C'$

Proof: below; Ex. 46, p. 579

PROOF Translation Theorem

A translation is an isometry.



GIVEN ▶ $P(a, b)$ and $Q(c, d)$ are two points on a figure translated by $(x, y) \rightarrow (x + s, y + t)$.

PROVE ▶ $PQ = P'Q'$

The translation maps $P(a, b)$ to $P'(a + s, b + t)$ and $Q(c, d)$ to $Q'(c + s, d + t)$.

Use the Distance Formula to find PQ and $P'Q'$. $PQ = \sqrt{(c - a)^2 + (d - b)^2}$.

$$P'Q' = \sqrt{[(c + s) - (a + s)]^2 + [(d + t) - (b + t)]^2}$$

$$= \sqrt{(c + s - a - s)^2 + (d + t - b - t)^2}$$

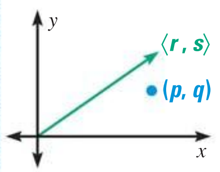
$$= \sqrt{(c - a)^2 + (d - b)^2}$$

Therefore, $PQ = P'Q'$ by the Transitive Property of Equality.

VECTORS Another way to describe a translation is by using a vector. A **vector** is a quantity that has both direction and *magnitude*, or size. A vector is represented in the coordinate plane by an arrow drawn from one point to another.

USE NOTATION

Use brackets to write the component form of the vector $\langle r, s\rangle$. Use parentheses to write the coordinates of the point (p, q).

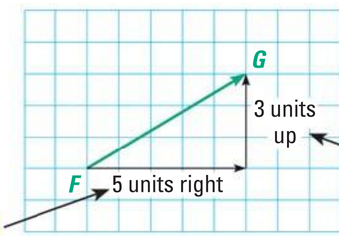


KEY CONCEPT *For Your Notebook*

Vectors

The diagram shows a vector named $\overrightarrow{FG}$, read as "vector *FG*."

The **initial point**, or starting point, of the vector is *F*.



The **terminal point**, or ending point, of the vector is *G*.

vertical component

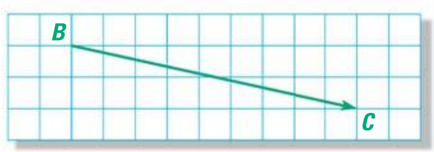
horizontal component

The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{FG}$ is $\langle 5, 3\rangle$.

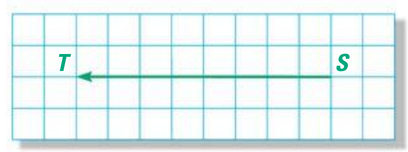
EXAMPLE 3 Identify vector components

Name the vector and write its component form.

a.



b.



Solution

a. The vector is $\overrightarrow{BC}$. From initial point *B* to terminal point *C*, you move 9 units right and 2 units down. So, the component form is $\langle 9, -2\rangle$.

b. The vector is $\overrightarrow{ST}$. From initial point *S* to terminal point *T*, you move 8 units left and 0 units vertically. The component form is $\langle -8, 0\rangle$.

EXAMPLE 4 Use a vector to translate a figure

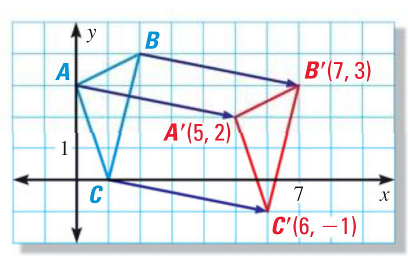
The vertices of $\triangle ABC$ are $A(0, 3)$, $B(2, 4)$, and $C(1, 0)$. Translate $\triangle ABC$ using the vector $\langle 5, -1\rangle$.

USE VECTORS

Notice that the vector can have different initial points. The vector describes only the direction and magnitude of the translation.

Solution

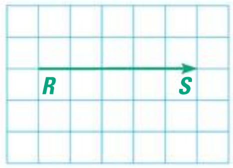
First, graph $\triangle ABC$. Use $\langle 5, -1\rangle$ to move each vertex 5 units to the right and 1 unit down. Label the image vertices. Draw $\triangle A'B'C'$. Notice that the vectors drawn from preimage to image vertices are parallel.



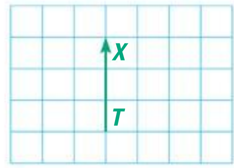
✓ GUIDED PRACTICE for Examples 3 and 4

Name the vector and write its component form.

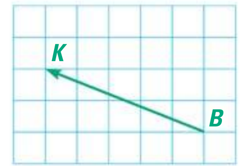
4.



5.



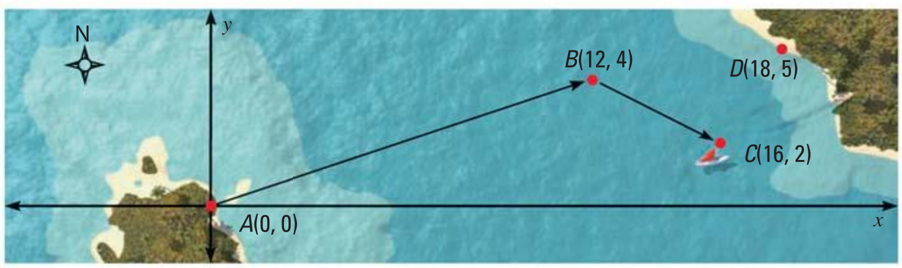
6.



7. The vertices of $\triangle LMN$ are $L(2, 2)$, $M(5, 3)$, and $N(9, 1)$. Translate $\triangle LMN$ using the vector $\langle -2, 6 \rangle$.

EXAMPLE 5 Solve a multi-step problem

NAVIGATION A boat heads out from point A on one island toward point D on another. The boat encounters a storm at B, 12 miles east and 4 miles north of its starting point. The storm pushes the boat off course to point C, as shown.



a. Write the component form of $\overrightarrow{AB}$.

b. Write the component form of $\overrightarrow{BC}$.

c. Write the component form of the vector that describes the straight line path from the boat's current position C to its intended destination D.

Solution

a. The component form of the vector from $A(0, 0)$ to $B(12, 4)$ is

$$\overrightarrow{AB} = \langle 12 - 0, 4 - 0 \rangle = \langle 12, 4 \rangle.$$

b. The component form of the vector from $B(12, 4)$ to $C(16, 2)$ is

$$\overrightarrow{BC} = \langle 16 - 12, 2 - 4 \rangle = \langle 4, -2 \rangle.$$

c. The boat is currently at point C and needs to travel to D. The component form of the vector from $C(16, 2)$ to $D(18, 5)$ is

$$\overrightarrow{CD} = \langle 18 - 16, 5 - 2 \rangle = \langle 2, 3 \rangle.$$

✓ GUIDED PRACTICE for Example 5

8. **WHAT IF?** In Example 5, suppose there is no storm. Write the component form of the vector that describes the straight path from the boat's starting point A to its final destination D.

9.1 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 35

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, and 42

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A _?_ is a quantity that has both _?_ and magnitude.

2. ★ **WRITING** *Describe* the difference between a vector and a ray.

EXAMPLE 1 on p. 572 for Exs. 3–10

IMAGE AND PREIMAGE **Use the translation $(x, y) \rightarrow (x - 8, y + 4)$.**

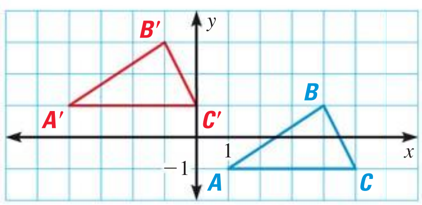
3. What is the image of $A(2, 6)$?
4. What is the image of $B(-1, 5)$?
5. What is the preimage of $C'(-3, -10)$?
6. What is the preimage of $D'(4, -3)$?

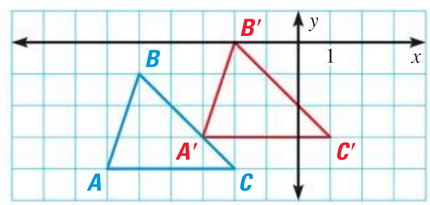
GRAPHING AN IMAGE **The vertices of $\triangle PQR$ are $P(-2, 3)$, $Q(1, 2)$, and $R(3, -1)$. Graph the image of the triangle using prime notation.**

7. $(x, y) \rightarrow (x + 4, y + 6)$
8. $(x, y) \rightarrow (x + 9, y - 2)$
9. $(x, y) \rightarrow (x - 2, y - 5)$
10. $(x, y) \rightarrow (x - 1, y + 3)$

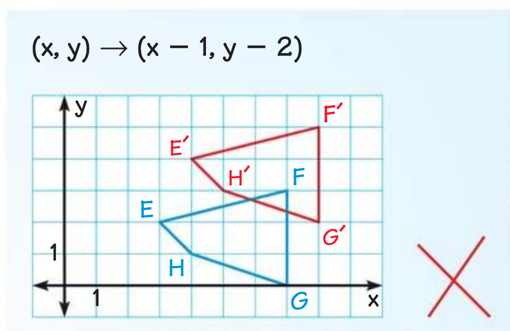
EXAMPLE 2 on p. 573 for Exs. 11–14

WRITING A RULE **$\triangle A'B'C'$ is the image of $\triangle ABC$ after a translation. Write a rule for the translation. Then *verify* that the translation is an isometry.**

11.


12.


13. **ERROR ANALYSIS** *Describe* and correct the error in graphing the translation of quadrilateral *EFGH*.

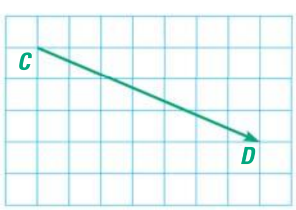


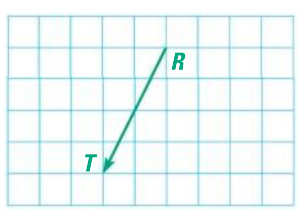
14. ★ **MULTIPLE CHOICE** Translate $Q(0, -8)$ using $(x, y) \rightarrow (x - 3, y + 2)$.

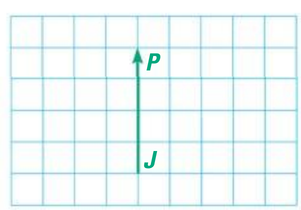
(A) $Q'(-2, 5)$ (B) $Q'(3, -10)$ (C) $Q'(-3, -6)$ (D) $Q'(2, -11)$

EXAMPLE 3 on p. 574 for Exs. 15–23

IDENTIFYING VECTORS **Name the vector and write its component form.**

15.


16.


17.


VECTORS **Use the point $P(-3, 6)$. Find the component form of the vector that describes the translation to P'.**

18. $P'(0, 1)$ **19.** $P'(-4, 8)$ **20.** $P'(-2, 0)$ **21.** $P'(-3, -5)$

TRANSLATIONS **Think of each translation as a vector. *Describe* the vertical component of the vector. *Explain*.**

22.

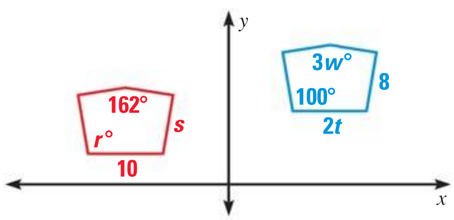

23.

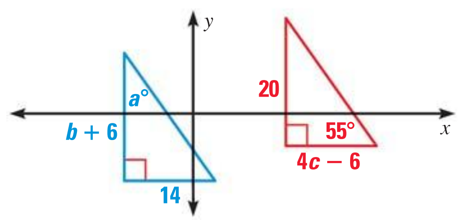

EXAMPLE 4
on p. 574
for Exs. 24–27

TRANSLATING A TRIANGLE **The vertices of $\triangle DEF$ are $D(2, 5)$, $E(6, 3)$, and $F(4, 0)$. Translate $\triangle DEF$ using the given vector. Graph $\triangle DEF$ and its image.**

24. $\langle 6, 0\rangle$ **25.** $\langle 5, -1\rangle$ **26.** $\langle -3, -7\rangle$ **27.** $\langle -2, -4\rangle$

xy ALGEBRA **Find the value of each variable in the translation.**

28.


29.


30. **xy ALGEBRA** Translation A maps (x, y) to $(x + n, y + m)$. Translation B maps (x, y) to $(x + s, y + t)$.

a. Translate a point using Translation A, then Translation B. Write a rule for the final image of the point.

b. Translate a point using Translation B, then Translation A. Write a rule for the final image of the point.

c. *Compare* the rules you wrote in parts (a) and (b). Does it matter which translation you do first? *Explain*.

31. **MULTI-STEP PROBLEM** The vertices of a rectangle are $Q(2, -3)$, $R(2, 4)$, $S(5, 4)$, and $T(5, -3)$.

a. Translate $QRST$ 3 units left and 2 units down. Find the areas of $QRST$ $Q'R'S'T'$.

b. *Compare* the areas. Make a conjecture about the areas of a preimage and its image after a translation.

32. **CHALLENGE** The vertices of $\triangle ABC$ are $A(2, 2)$, $B(4, 2)$, and $C(3, 4)$.

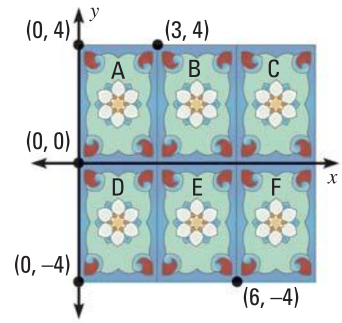
a. Graph the image of $\triangle ABC$ after the transformation $(x, y) \to (x + y, y)$. Is the transformation an isometry? *Explain*. Are the areas of $\triangle ABC$ and $\triangle A'B'C'$ the same?

b. Graph a new triangle, $\triangle DEF$, and its image after the transformation given in part (a). Are the areas of $\triangle DEF$ and $\triangle D'E'F'$ the same?

PROBLEM SOLVING

EXAMPLE 2
on p. 573
for Exs. 33–34

HOME DESIGN **Designers can use computers to make patterns in fabrics or floors. On the computer, a copy of the design in Rectangle A is used to cover an entire floor. The translation $(x, y) \to (x + 3, y)$ maps Rectangle A to Rectangle B.**



33. Use coordinate notation to describe the translations that map Rectangle A to Rectangles C, D, E, and F.

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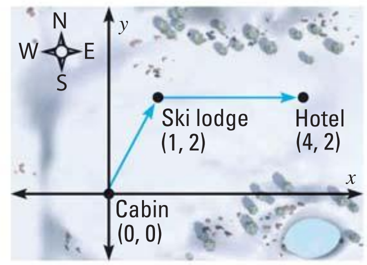
34. Write a rule to translate Rectangle F back to Rectangle A.

@HomeTutor for problem solving help at classzone.com

EXAMPLE 5
on p. 575
for Exs. 35–37

SNOWSHOEING **You are snowshoeing in the mountains. The distances in the diagram are in miles. Write the component form of the vector.**

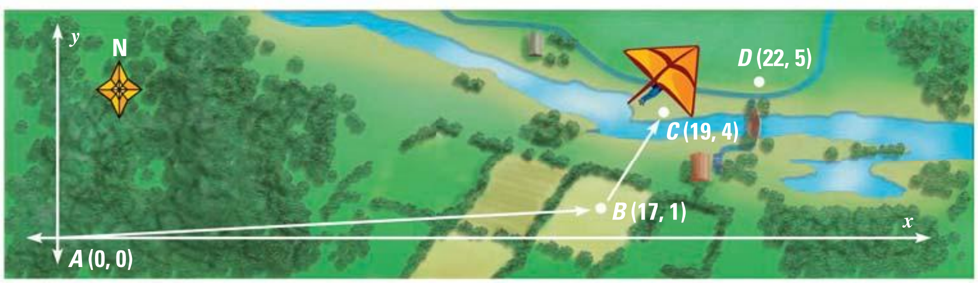


35. From the cabin to the ski lodge

36. From the ski lodge to the hotel

37. From the hotel back to your cabin

HANG GLIDING **A hang glider travels from point A to point D. At point B, the hang glider changes direction, as shown in the diagram. The distances in the diagram are in kilometers.**



38. Write the component form for $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

39. Write the component form of the vector that describes the path from the hang glider's current position C to its intended destination D.

40. What is the total distance the hang glider travels?

41. Suppose the hang glider went straight from A to D. Write the component form of the vector that describes this path. What is this distance?

42. ★ **EXTENDED RESPONSE** Use the equation $2x + y = 4$.

a. Graph the line and its image after the translation $\langle -5, 4 \rangle$. What is an equation of the image of the line?

b. *Compare* the line and its image. What are the slopes? the y-intercepts? the x-intercepts?

c. Write an equation of the image of $2x + y = 4$ after the translation $\langle 2, -6 \rangle$ *without* using a graph. *Explain* your reasoning.

○ = WORKED-OUT SOLUTIONS on p. WS1

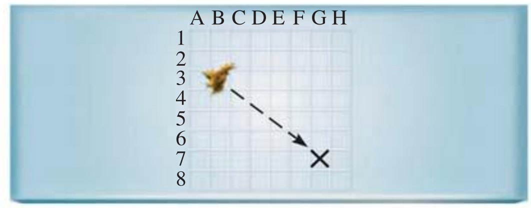
★ = STANDARDIZED TEST PRACTICE

43. **SCIENCE** You are studying an amoeba through a microscope. Suppose the amoeba moves on a grid-indexed microscope slide in a straight line from square B3 to square G7.

a. *Describe* the translation.

b. Each grid square is 2 millimeters on a side. How far does the amoeba travel?

c. Suppose the amoeba moves from B3 to G7 in 24.5 seconds. What is its speed in millimeters per second?



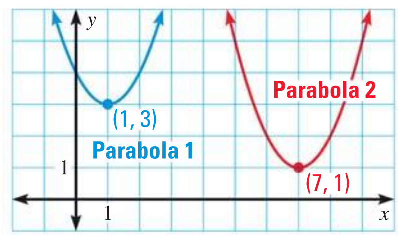
44. **MULTI-STEP PROBLEM** You can write the equation of a parabola in the form $y = (x - h)^2 + k$, where (h, k) is the *vertex* of the parabola. In the graph, an equation of Parabola 1 is $y = (x - 1)^2 + 3$, with vertex (1, 3). Parabola 2 is the image of Parabola 1 after a translation.

a. Write a rule for the translation.

b. Write an equation of Parabola 2.

c. Suppose you translate Parabola 1 using the vector $\langle -4, 8\rangle$. Write an equation of the image.

d. An equation of Parabola 3 is $y = (x + 5)^2 - 3$. Write a rule for the translation of Parabola 1 to Parabola 3. *Explain* your reasoning.



45. **TECHNOLOGY** The standard form of an exponential equation is $y = a^x$, where $a > 0$ and $a \neq 1$. Use the equation $y = 2^x$.

a. Use a graphing calculator to graph $y = 2^x$ and $y = 2^x - 4$. *Describe* the translation from $y = 2^x$ to $y = 2^x - 4$.

b. Use a graphing calculator to graph $y = 2^x$ and $y = 2^{x-4}$. *Describe* the translation from $y = 2^x$ to $y = 2^{x-4}$.

46. **CHALLENGE** Use properties of congruent triangles to prove part of Theorem 9.1, that a translation preserves angle measure.

MIXED REVIEW

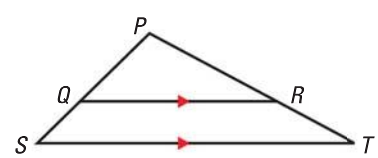
PREVIEW
Prepare for Lesson 9.2 in Exs. 47–50.

Find the sum, difference, product, or quotient. *(p. 869)*

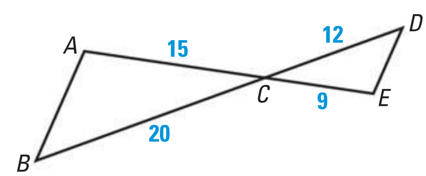
47. $-16 - 7$ 48. $6 + (-12)$ 49. $(13)(-2)$ 50. $16 \div (-4)$

Determine whether the two triangles are similar. If they are, write a similarity statement. *(pp. 381, 388)*

51.



52.



Points *A, B, C,* and *D* are the vertices of a quadrilateral. Give the most specific name for *ABCD*. *Justify* your answer. *(p. 552)*

53. $A(2, 0)$, $B(7, 0)$, $C(4, 4)$, $D(2, 4)$

54. $A(3, 0)$, $B(7, 2)$, $C(3, 4)$, $D(1, 2)$